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Spinning Charged Solutions in 2+1 Dimensional Einstein-Maxwell-Dilaton Gravity

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Abstract

We report a new class of rotating charged solutions in 2+1 dimensions. These solutions are obtained for Einstein-Maxwell gravity coupled to a dilaton field with selfdual electromagnetic fields. The mass and the angular momentum of these solutions computed at spatial infinity are finite. The class of solutions considered here have naked singularities and are asymptotically flat.

1 Introduction

Interest in 2+1 dimensional gravity has been heighten by the discovery of a black hole solution by Bañados *et al.* [1]. This black hole, (named BTZ) has anti-de Sitter structure locally and globally differ to anti-de Sitter by identifications done with a discrete subgroup of the isometry group of anti-de Sitter space, $SO(2,2)$ [2]. It enjoys many black hole properties of its counterparts in higher dimensions which makes BTZ a suitable model to understand black hole physics in a technically simpler setting.

Extension of the BTZ black hole with charge have been met with mixed success. The first investigation into static charged black holes was done by Bañados *et al.*[1]. Due to the logarithmic nature of the electromagnetic potential, these solutions gave rise to unphysical properties [3]. The horizonless static solution with magnetic charge were studied by Hirshmann *et al.*[3], and the persistence of these unphysical properties was highlighted by Chan[4]. Kamata *et al.*[5] presented a rotating charged black hole with self (anti-self) duality imposed on the electromagnetic fields. The resulting solutions were asymptotic to an extreme BTZ black hole solution but had diverging mass and angular momentum [4]. Clément[6], Fernando and Mansouri[7], introduced a Chern-Simons term as a regulator to screen the electromagnetic potential and obtained horizonless charged solutions.

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In this work, we couple a dilaton to Einstein-Maxwell gravity to obtain rotating charged solutions with finite mass and finite angular momentum. It is well known that the introduction of a dilaton field drastically changes the space-time structure in 3+1 dimensions. Furthermore, studying dilaton gravity is important since it arises in low energy string theory. Therefore it is worthwhile to see how the presence of the dilaton would modify the BTZ black hole solutions and how it would help in curing the divergences occurred in the previous charged solutions.

Einstein-Maxwell-Dilaton action in 2+1 dimensions is written as follows:

$$I = \int d^3x\sqrt{|g|}\left[R - 2\Lambda e^{\beta\phi} - \frac{\gamma}{2}(\bigtriangledown^\mu\phi \bigtriangledown_\mu \phi) - e^{-4\alpha\phi}F^2\right] \tag{1}$$

Here Λ is the cosmological constant and we consider the case for $\Lambda < 0$ which corresponds to anti-de Sitter spaces. β, γ, α are coupling constant. In this expression $1/16\pi G$ is taken to be 1. The above action is the low energy string action when $\gamma = 8$, $\beta = 4$, $\alpha = 1$. There are several work related to charged solutions to the above action. Chan *et al.*[8] have studied a one parameter family of static charged dilaton black holes which were non-asymptotically anti-de sitter and solutions with cosmological horizons when $4\alpha = \beta$. Park *et. al.*[9] obtained axially symmetric static solutions by using a dimensional reduction method. A more general magnetically charged solution to dilaton gravity was found by Koikawa *et al.*[10]. By applying T-duality to the static electric charged black holes of Chan *et al.*[14], Chen[11] obtained rotating charged black hole solutions to Einstein-Maxwell-Dilaton gravity. However, these solutions are not the most general rotating charged solutions to Einstein-Maxwell-Dilaton gravity.

The aim of the present paper is to search for rotating charged solutions to Einstein-Maxwell-Dilaton gravity with self duality imposed on the electromagnetic fields. The plan of the paper is follows. In section 2 we will compute the most general solutions with self duality imposed. In section 3 we will impose restrictions on coupling constants so that the mass and angular momentum are finite. In section 4 we will discuss the properties of theses solutions and finally give concluding remarks.

2 General Solutions

By extremizing the Lagrangian of the action in equation(1) with respect to the metric $g_{\mu\nu}$, the electromagnetic potential A_μ and the dilaton field ϕ, one obtains the corresponding field equations for gravitational, electromagnetic and dilaton respectively as follows.

$$R_{\mu\nu} = -2\Lambda g_{\mu\nu}e^{\beta\phi} + e^{-4\alpha\phi}(2F_{\mu\rho}F^\rho_\nu - g_{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma}) + \frac{\gamma}{2}(\bigtriangledown_\mu\phi)(\bigtriangledown_\nu\phi) \tag{2}$$

$$\bigtriangledown_\mu\,(e^{-4\alpha\phi}F^{\mu\nu}) = 0 \tag{3}$$

$$\frac{\gamma}{2}\bigtriangledown^\mu\bigtriangledown_\mu\phi + 2\alpha e^{-4\alpha\phi}F^2 - \beta e^{\beta\phi}\Lambda = 0 \tag{4}$$

Let us assume that the three dimensional space-time is stationary and circularly symmetric, having two commuting Killing vectors, $\frac{\partial}{\partial \phi}$ and $\frac{\partial}{\partial t}$. Such a space-time could be parameterized with a line element as follows:

$$ds^2 = -N^2 dt^2 + L^{-2} d\rho^2 + K^2 (d\theta + N^\theta dt)^2 \tag{5}$$

The functions N, L, K and N^θ depends only on radial coordinate ρ. We use the tetrad formalism and Cartan structure equations to look for solutions. Obvious non coordinate basis for the above metric would be,

$$e^0 = N dt; \qquad e^1 = K(d\theta + N^\theta dt); \qquad e^2 = L^{-1} d\rho \tag{6}$$

The indices $a, b = 0, 1, 2$ are for the orthonormal basis and $\mu, \nu = 0, 1, 2$ for the coordinate basis with $x^0 = t$; $x^1 = \theta$; $x^2 = \rho$. The non-vanishing components of the electromagnetic field tensor in the coordinate basis are given by $F_{t\rho} = E$, $F_{\rho\theta} = B$, and in the orthonormal basis they are given by $F_{02} = \hat{E}$, $F_{21} = \hat{B}$. They are related by,

$$E = \frac{(\hat{E}N - \hat{B}KN^\theta)}{L}; \qquad B = \frac{\hat{B}K}{L} \tag{7}$$

From electromagnetic field equations(3),

$$F^{t\rho} = \frac{C_1 e^{4\alpha\phi}}{\sqrt{|g|}}; \qquad F^{r\theta} = \frac{C_2 e^{4\alpha\phi}}{\sqrt{|g|}} \tag{8}$$

In order to seek solutions to the gravitational field equations, we make the ansatz that, the electric and the magnetic fields are self dual in the orthonormal basis. Therefore,

$$\hat{E} = \hat{B} = u(\rho) \tag{9}$$

With the use of equations(7,8,9),

$$u(\rho) = -C_1 \frac{\exp[4\alpha\phi]}{K}$$

$$N^\theta = \frac{N}{K} - \frac{C_2}{C_1} \tag{10}$$

Considering the behavior of $u = \hat{E}$ for flat space-times, we can represent C_1 with electric charge Q_e. A value for C_2 will be assigned later.

Having used the ansatz of self duality, the gravitational field equations in the orthonormal basis takes the following form for a circularly symmetric space time.

$$R_{00} = L^2 \left(\frac{N''}{N} + \frac{N'K'}{NK} \right) + \frac{LL'N'}{N} - 2 \left(\frac{KLN^{\theta\prime}}{2N} \right)^2 = -2\Lambda e^{\beta\phi} + 2e^{-4\alpha\phi} u^2 \tag{11}$$

$$R_{11} = -L^2\left(\frac{K''}{K} + \frac{N'K'}{NK}\right) - \frac{LL'K'}{K} - 2\left(\frac{KLN^{\theta'}}{2N}\right)^2 = 2\Lambda e^{\beta\phi} + 2e^{-4a\phi}u^2 \qquad (12)$$

$$R_{01} = -\frac{L}{K^2}\left(\frac{K^3LN^{\theta'}}{2N}\right)' = -2e^{-4\alpha\phi}u^2 \qquad (13)$$

$$G_{22} = L^2\left(\frac{N'K'}{NK}\right) + \left(\frac{KLN^{\theta'}}{2N}\right)^2 = -\Lambda e^{\beta\phi} + \frac{\gamma}{4}(\hat{\bigtriangledown}_2\phi)^2 \qquad (14)$$

The gravitational field equations(11-14), the dilaton field equation(4) and the conditions (10) leads to the final equations to be solved as follows:

$$(LX')' = -4\Lambda e^{\beta\phi}\frac{X}{L} \qquad (15)$$

$$L(HK^2)' = 2Q_e^2 e^{4\alpha\phi} \qquad (16)$$

$$\left(\frac{X'L}{2X}\right)^2 = \frac{\gamma}{4}(\phi' L)^2 - \Lambda e^{\beta\phi} \qquad (17)$$

$$\gamma(\phi LX)' = 2\Lambda\beta e^{\beta\phi}\frac{X}{L} \qquad (18)$$

where,

$$X = NK; \qquad H = \frac{L}{2}\left(\frac{N'}{N} - \frac{K'}{K}\right) \qquad (19)$$

Here, the prime is given by $d/d\rho$. Equation(15) is obtained by observing that $R_{00} + 2\Lambda e^{\beta\phi} = R_{11} - 2\Lambda e^{\beta\phi}$. Equation(16) is obtained by the fact that $R_{00} + R_{11} = -2R_{01}$. Equation(17) is just equation(14) rewritten. Equation(18) is the dilaton field equations with the self duality imposed. The solution to the above set of equations are,

$$X = a_1 exp(\frac{-2\gamma\phi}{\beta})$$

$$L = \frac{exp(\frac{\beta\phi}{2})}{b_0\phi'}$$

$$K^2 = a_2 exp(\frac{-2\gamma\phi}{\beta}) - \frac{2b_0a_3}{(\frac{2\gamma}{\beta} - \frac{\beta}{2})}exp(\frac{-\beta\phi}{2}) - \frac{4Q_e^2b_0^2}{(4\alpha - \frac{\beta}{2})(4\alpha - \beta + \frac{2\gamma}{\beta})}exp((4\alpha - \beta)\phi) \qquad (20)$$

Here $b_0^2 = \frac{(4\gamma^2 - \gamma\beta^2)}{(4|\Lambda|\beta^2)}$ and the above solutions are valid only when $b_0^2 > 0$, $(4\alpha - \frac{\beta}{2}) \neq 0$, $(4\alpha - \beta + \frac{2\gamma}{\beta}) \neq 0$, and $\beta \neq 0$. Here a_1, a_2, a_3 are integrating constants.

From the above equations it is clear that the all the functions N, L, K, N^θ depend on the dilaton field ϕ. In order to compare these solutions with the previously obtained

dilaton solutions and also to see the correspondence with the BTZ black hole we pick L to be the following.

$$L = \frac{|\Lambda|^{1/2}(\rho^2 - \rho_0^2)^{\omega}}{\rho} \tag{21}$$

Here, ω and ρ_0^2 are constants which would be determined by imposing some restrictions on the solutions to give finite mass and finite angular momentum. Note that equation(21) is equivalent to the dilaton field ϕ and X been,

$$\phi = ln\left(b_1(\rho^2 - \rho_0^2)^{\frac{2(\omega-1)}{\beta}}\right); \qquad X = |\Lambda|^{1/2}(\rho^2 - \rho_0^2)^{\frac{4\gamma(1-\omega)}{\beta^2}} \tag{22}$$

where $b_1 = [\frac{-\beta}{4b_0(-\omega+1)|\Lambda|^{1/2}}]^{-2/\beta}$ and a_1 is normalized so that $a_1 b_1^{\frac{-2\gamma}{\beta}} = |\Lambda|^{1/2}$. By substituting ϕ into equation(20),

$$K^2 = \bar{a}_2(\rho^2 - \rho_0^2)^{\frac{4\gamma(-\omega+1)}{\beta^2}} + \overline{a}_3(\rho^2 - \rho_0^2)^{(-\omega+1)} + \overline{Q}_e^2(\rho^2 - \rho_0^2)^{\frac{2(4\alpha-\beta)(\omega-1)}{\beta}} \tag{23}$$

where $\overline{a}_2 = a_2 b_1^{-2\gamma/\beta}$, $\overline{a}_3 = \frac{-2b_0 b_1^{(-\beta/2)} a_3}{(\frac{2\gamma}{\beta} - \frac{\beta}{2})}$, $\overline{Q}_e^2 = \frac{-4Q_e^2 b_0^2 b_1^{4\alpha-\beta}}{(4\alpha - \frac{\beta}{2})(4\alpha - \beta + \frac{2\gamma}{\beta})}$. We restrict $\omega \neq 1$. Otherwise the dilaton would be a constant and the solutions would be trivial.

3 Quasilocal Mass and Angular Momentum

In this section, we will impose restrictions on the values of $\alpha, \beta, \gamma, \omega$ so that the above solutions have finite mass and finite angular momentum. Also we will interpret the integrating constants a_1, a_2, a_3 as appropriate physical constants. Here we will adopt the prescription of Brown *et.al.* [12] [13] in computing mass and angular momentum.

3.1 Quasilocal Angular Momentum

The quasilocal angular momentum according to the prescription given by Brown *et.al.*[12] [13] is,

$$J(\rho) = \frac{K^3 N^{\theta'} L}{N} \tag{24}$$

For the metric in equation(20) it is equivalent to,

$$J(\rho) = -2HK^2 \tag{25}$$

From equations(19) and (23),

$$K^2 H = -|\Lambda|^{1/2}\overline{a}_3 - 2|\Lambda|^{\frac{1}{2}}\overline{Q}_e^2(\frac{4\alpha}{\beta} - 1)(\omega - 1)(\rho^2 - \rho_0^2)^{\frac{(8\alpha-\beta)(\omega-1)}{\beta}} \tag{26}$$

Hence for large ρ, $J(\rho)$ becomes finite only if

$$(\omega-1)(\frac{8\alpha}{\beta}-1)\leq 0 \tag{27}$$

Since $\omega\neq 1$ and $(8\alpha-\beta)\neq 0$, the present class of solutions will obey strictly smaller condition for equation(27). Hence,

$$\lim_{\rho\to\infty} J(\rho)=J=2|\Lambda|^{\frac{1}{2}}\overline{a}_3 \tag{28}$$

3.2 Quasilocal Mass

From the definition of Brown *et. al.* [12][13], the quasilocal mass is given by,

$$M(\rho)=2N(\rho)\left[L_0(\rho)-L(\rho)\right])-J(\rho)N^\theta(\rho) \tag{29}$$

Here, $L_0(\rho)$ is chosen to be the reference when there is zero mass. In comparison with the BTZ black hole we choose

$$L_0=(L)_{\rho_0=0}=|\Lambda|^{1/2}\frac{\rho^{2\omega}}{\rho} \tag{30}$$

Consider the first term in equation(29),

$$2N(L_0-L)=\frac{2|\Lambda|^{\frac{1}{2}}X}{K}\left(\omega\rho_0^2\rho^{(2\omega-3)}+\text{lower order terms of }\rho\right) \tag{31}$$

For finite mass, (X/K) should behave as ρ^n with $n\leq-(2\omega-3)$. If $n<-(2\omega-3)$, then this solution will correspond to the "massless" BTZ solution for appropriate limits. Hence to avoid such extreme cases, we would pick n to be $(-2\omega+3)$. Therefore, since at large ρ,

$$X\to\rho^{\frac{8\gamma(-\omega+1)}{\beta^2}} \tag{32}$$

K should behave as,

$$K\to\rho^{(\frac{8\gamma(-\omega+1)}{\beta^2}+(2\omega-3))} \tag{33}$$

for large ρ for $n=(-2\omega+3)$. Now, we will consider N^θ term at large ρ.

$$N^\theta=\frac{X}{K^2}-\frac{C_2}{C_1} \tag{34}$$

To impose the boundary condition $N^\theta(\infty)=0$, X/K^2 has to converge for $\rho\to\infty$. With the constraint in equation(33) it means,

$$(-\omega+1)(-\frac{8\gamma}{\beta^2}+4)+2\leq 0 \tag{35}$$

To approximate the behavior of K^2 to be ρ^2 at large and small ρ, we assume all terms in K^2 has positive powers of ρ^2. Considering the fact that $\overline{a}_3(\rho^2-\rho_0^2)^{(-\omega+1)}$ term in K^2 is proportional to the angular momentum, to take appropriate limits, we take $\bar{a}_2(\rho^2-\rho_0^2)^{\frac{4\gamma(-\omega+1)}{\beta^2}}$ term to be the dominant power of K^2. Without loss of generality we assume $\overline{a}_2=1$. Hence

$$\frac{4\gamma(-\omega+1)}{\beta^2} > (-\omega+1) > 0$$

$$\frac{2(4\alpha-\beta)(\omega-1)}{\beta} > 0 \tag{36}$$

and from equation(33) and assumptions in equation(36), $\omega=\frac{(4\gamma-3\beta^2)}{(4\gamma-2\beta^2)}$. With all these preliminaries,

$$\lim_{\rho\to\infty} N^\theta(\rho) = (|\Lambda|^{1/2} - C_2/C_1) \tag{37}$$

To impose $N^\theta(\infty)=0$, we let $C_2=|\Lambda|^{1/2}C_1$. Hence,

$$\lim_{\rho\to\infty} M(\rho) = M = 2|\Lambda|\omega\rho_0^2 \tag{38}$$

leading to $\rho_0^2=\frac{M}{2|\Lambda|\omega}$.

4 Exact Solutions with Finite Mass and Finite Angular momentum

With the above preliminaries, the final form of the solutions with finite mass and finite angular momentum is,

$$L = \frac{|\Lambda|^{1/2}(\rho^2-\rho_0^2)^{\omega}}{\rho}$$

$$N = \frac{|\Lambda|^{1/2}(\rho^2-\rho_0^2)^{(-2\omega+3)}}{K}$$

$$N^\theta = \frac{N}{K} - |\Lambda|^{1/2}$$

$$K^2 = (\rho^2-\rho_0^2)^{(-2\omega+3)} + \overline{a}_3(\rho^2-\rho_0^2)^{(-\omega+1)} + \overline{Q}_e^2(\rho^2-\rho_0^2)^{\frac{2(4\alpha-\beta)(\omega-1)}{\beta}} \tag{39}$$

$$\phi = ln\left(b_1(\rho^2-\rho_0^2)^{\frac{2(\omega-1)}{\beta}}\right) \tag{40}$$

With $\omega=\frac{(4\gamma-3\beta^2)}{(4\gamma-2\beta^2)}$, $\rho_0^2=\frac{M}{2|\Lambda|\omega}$ and $\overline{a}_3=\frac{J}{2|\Lambda|^{\frac{1}{2}}}$. The electromagnetic potential is

$$A = A_\mu dx^\mu = \frac{(\rho^2-\rho_0^2)^m}{2m}\left(|\Lambda|^{1/2}dt + d\theta\right). \tag{41}$$

where $m = \frac{\beta(8\alpha-\beta)}{2(\beta^2-2\gamma)}$. From the constraint imposed in equation(27), it is obvious that $m < 0$. Hence, the potential is finite at large ρ. Therefore, the presence of the dilaton "screens" the electromagnetic potential. To clarify this further, if we look at dilaton field in flat space for self-dual case as considered above, the values for $\hat{E}$ and $\hat{B}$ would be $Q_e exp[4\alpha b_0]/\rho$. Hence the potential $A_t = Q_e r^{4\alpha b_0}/(4\alpha b_0)$ is finite for $4\alpha b_0 < 0$. Therefore the presence of the dilaton modifies the Coulomb force in 2+1 dimensions. We may recall that in [6][7], a topological mass term $m_p \epsilon^{\alpha\beta\gamma} A_\alpha F_{\beta\gamma}$ was introduced to Einstein-Maxwell gravity to cure the divergences of the quasilocal mass M which gives a similar effect to the Coulomb force. If we compare the electric field in three separate cases,

$$E_{Coulomb} = \frac{Q_e}{\rho} \qquad E_{Dilaton} = \frac{Q_e \rho^{4\alpha b_0}}{\rho} \qquad E_{topological} = \frac{Q_e e^{-m_p \rho}}{\rho} \tag{42}$$

the topological mass term has a better regulating effect in comparison with the dilaton field.

4.1 Causal Structure

The curvature scalars in 2+1 dimensions are R, $R_{\alpha\beta}R^{\alpha\beta}$ and $detR_{\alpha\beta}/detg_{\alpha\beta}$. For the above solutions, with $\gamma, \beta, \alpha \neq 0$ and for $M > 0$, all of them diverge at $\rho = \rho_0$ and finite everywhere else. Hence the curvature singularity at $\rho = \rho_0$ is a naked singularity without horizons. However, K^2 which is the $g_{\theta\theta}$ term in the metric has to be positive to avoid closed time like curves since θ is a periodic coordinate. Even with the constraints imposed on the parameters α, β, γ, there is a possibility that K^2 would become negative. Hence one has to include the possibility of closed time like curves for these solutions. The scalar curvature R is $(6\Lambda + \frac{\gamma}{4b_0^2})(\rho^2 - \rho_0^2)^{\frac{(\omega-1)}{2}}$. Since $(1-\omega) > 0$, $R \rightarrow 0$ for large ρ. Therefore the solution becomes flat asymptotically.

4.2 The relation with BTZ Black Hole

Note that above discussion is for when γ, β and α is non vanishing. To see the correspondence of the above solutions with BTZ black hole, let us take the limit $\omega \rightarrow 1$ and $Q_e \rightarrow 0$. Then,

$$M \rightarrow 2|\Lambda|\rho_0^2; \qquad \phi \rightarrow 0; \qquad K^2 \rightarrow (\rho^2 - \rho_0^2) + \overline{a}_3 \tag{43}$$

If $\rho_0^2 = \overline{a}_3$ then $M = |\Lambda|^{1/2} J$ with the following metric,

$$L = N \rightarrow \frac{|\Lambda|^{1/2}(\rho^2 - \frac{M}{2|\Lambda|^{1/2}})}{\rho}$$

$$K \rightarrow \rho$$

$$N^\theta \to \frac{J}{2\rho^2} \tag{44}$$

Hence the solution obtained in this work approaches an extreme BTZ black hole as a special case. The presence of the dilaton and charge have left the BTZ space-time horizonless and asymptotically flat.

5 Conclusions

We have obtained a family of rotating charged solutions to Einstein-Maxwell-Dilaton gravity in 2+1 dimensions. Here we have imposed self duality on the electric and magnetic field to facilitate solve the field equations exactly. With certain constraints on the coupling constants γ, β and α, we obtained solutions with finite mass and finite angular momentum. For non-zero values of α β and γ, the class of solutions considered in this paper are horizonless, have naked singularities and are asymptotically flat. These solutions approaches the extreme BTZ black hole solutions as a special case for $\alpha, \beta, \gamma \to 0$. The presence of the dilaton "screens" the electromagnetic potential and modifies the structure of the space-time considerably. However, since the metric depends on the dilaton as it is clear from equation(20), one may use other polynomial functions for the dilaton field in these solutions to understand how the space-time structure changes accordingly. It is also a question how a massive dilaton would effect the space-time structure of the above solutions. In extending this work it may be possible to include a potential of the form $V(\phi) = 2\Lambda_1 e^{\beta_1 \phi} + 2\Lambda_2 e^{\beta_2 \phi}$ to the action considered in this paper. These kind of potentials are investigated in dimensions $n \geq 4$ and have shown the possibility of having three horizons by Chan *et al.* [14]. It would be interesting to see whether one can construct rotating charged dilaton black holes with the above potential in 2+1 dimensions.

We may recall that Chen [11] obtained rotating charged black hole solutions for dilaton gravity with $\gamma = 4$ and $4\alpha = \beta$ by a T-duality transformation on the static charged black holes of Chan *et al.* [8]. The solutions obtained in this paper are for more general values of α, β, γ and they are not T-dual to the solutions of Chen [11]. Furthermore instead of Maxwell fields, one can include Yang-Mills fields to study the space-time structure for dilaton gravity in 2+1 dimensions which we hope to report elsewhere.

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